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# Heptagonal Graceful Labeling of Star Related Graphs

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## Abstract

Numbers of the form $\frac{5n^2-3n}{2}$ for all $n \geq 1$ are called heptagonal numbers. Let $G$ be a graph with $p$ vertices and $q$ edges. Let $f: V(G) \to \{0,1,2,\dots,N_q\}$ where $N_q$ is the $q^{th}$ heptagonal number be an injective function. Define the $f^*: E(G) \to \{1,7,18,\dots,N_q\}$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges $uv \in E(G)$. If $f^*(E(G))$ is a sequence of distinct consecutive numbers $N_1, N_2, \dots, N_q$ then the function $f$ is said to be heptagonal graceful labeling and the graph which admits such a labeling is called a heptagonal graceful graph. In this paper heptagonal graceful labeling of some graphs is studied.

**Keywords:** Graceful labeling, Graceful graphs, Polygonal graceful labeling, Heptagonal graceful number, Heptagonal graceful labeling, Heptagonal graceful graphs

## 1. Introduction

Graphs considered in this paper are finite, undirected and simple. Let $G = (V, E)$ be a graph with $p$ vertices and $q$ edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertex (edge/both) then the labeling is called a vertex (edge/both) labeling. Rosa (1966) introduced $\beta$-valuation of a graph. Golomb (1972) called it as a graceful labeling. Let $G$ be a $(p, q)$ graph. A one to one function $f: V(G) \to \{0,1,2,\dots,q\}$ is called a graceful labeling of $G$ if the induced edge labeling $f: V(G) \to \{0,1,2,\dots,q\}$ defined by $f(e) = |f(u) - f(v)|$ for each edges $e = uv$ of $G$ is also one to one. The graph $G$ possessing graceful labeling is called graceful graph. In Acharya (1982), certain families of graceful graphs were constructed. There are several types of graceful labeling and a detailed survey is found in Gallian (2018). Labeled graphs are becoming an increasing useful family of mathematical models for a board range of application like designing X-Ray crystallography, formulating a communication network addressing system, determining an optimal circuit layouts, problems in additive number theory etc. In this paper, heptagonal graceful labeling of some graphs is studied.

## 2. Materials and Methods

Let $f: V(G) \to \{0,1,2,\dots,N_q\}$ where $N_q$ is the $q^{th}$ heptagonal number be an injective function. Define the $f^*: E(G) \to \{1,7,18,\dots,N_q\}$ such that $f^*(uv) = |f(u) - (f(v)|$ for all edges $uv \in E(G)$. If $f^*(E(G))$ is a

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sequence of distinct consecutive numbers $N_1, N_2, \ldots, N_q$ then the function $f$ is said to be heptagonal graceful labeling and the graph which admits such a labeling is called a heptagonal graceful graph Golomb (1972). Here we discuss some of the basic definitions of graphs used in this manuscript.

### 2.1 Definition

Shurb $\mathrm{St}(n_1, n_2, \ldots, n_m)$ Mahendran (2021) is a graph obtained by connecting a vertex $v_0$ to the central vertex of each of $m$ numbers of stars.

### 2.2 Definition

Coconut tree $\mathrm{CT}(n, m)$ Mahendran (2021) is obtained by identifying the central vertex of $K_{1,n}$ with a pendant vertex of the path $P_m$.

### 2.3 Definition

Banana tree $\mathrm{Bt}(n_1, n_2, \ldots, n_m)$ (m times n) Mahendran (2021) is a graph obtained by connecting a vertex $v_0$ to one leaf of each of $m$ number of stars.

### 2.4 Definition

A complete bipartite graph $K_{1,n} \odot K_1$ is called a star Mahendran (2021) and it has $n + 1$ vertices and $n$ edges.

### 2.5 Definition

Y -tree Mahendran (2021) on $n + 1$ vertices, denoted by $Y_n$, is obtained from a path $P_n$ by attaching exactly a pendant vertices to the $(n-1)^{th}$ vertex of $P_n$

## 3. Results and Discussions

### 3.1 Theorem

Shurb $\mathrm{St}(n_1, n_2, \ldots, n_m)$ Mahendran (2021) is heptagonal graceful.

**Proof**

Let $G$ be the graph $\mathrm{St}(n_1, n_2, \ldots, n_m)$.Let $V(G) = \{v, v_i, v_{ij}: 1 \le i \le m, 1 \le j \le n_i\}$ and

$E(G) = \{vv_i, v_i v_{ij}: 1 \le i \le m, 1 \le j \le n_i\}$

$G$ has $m + n_1 + n_2 + \cdots + n_m + 1$ vertices and $m + n_1 + n_2 + \cdots + n_m$ edges.Let $q = m + n_1 + n_2 + \cdots + n_m$Let $f: V(G) \to \{0,1,2, \ldots, N_q\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = N_{q-[n_1+n_2+\cdots+n_{i-1}+i-1]}, 1 \le i \le m$

$f(v_{ij}) = N_{q-[n_1+n_2+\cdots+n_{i-1}+i-1]} - N_{q-[n_1+n_2+\cdots+n_{i-1}+j+i-1]}; 1 \le i \le m, 1 \le j \le n_i$

Let $f^*$ be the induced edge labeling of $f$

$f^*(vv_i) = N_{q-[n_1+n_2+\cdots+n_{i-1}+i-1]}, 1 \le i \le m$

$f^*(v_i v_{ij}) = N_{q-[n_1+n_2+\cdots+n_{i-1}+i-1]} - N_{q-[n_1+n_2+\cdots+n_{i-1}+j+i-1]}; 1 \le i \le m, 1 \le j \le n_i$

The induced edge labels $N_1, N_2, \ldots, N_q$ are distinct and consecutive heptagonal numbers.Hence the Shurb is heptagonal graceful.

*3.1.1 Illustration*

Heptagonal graceful labeling of $\mathrm{St}(2,3,4,5)$ is given in Fig. 1;

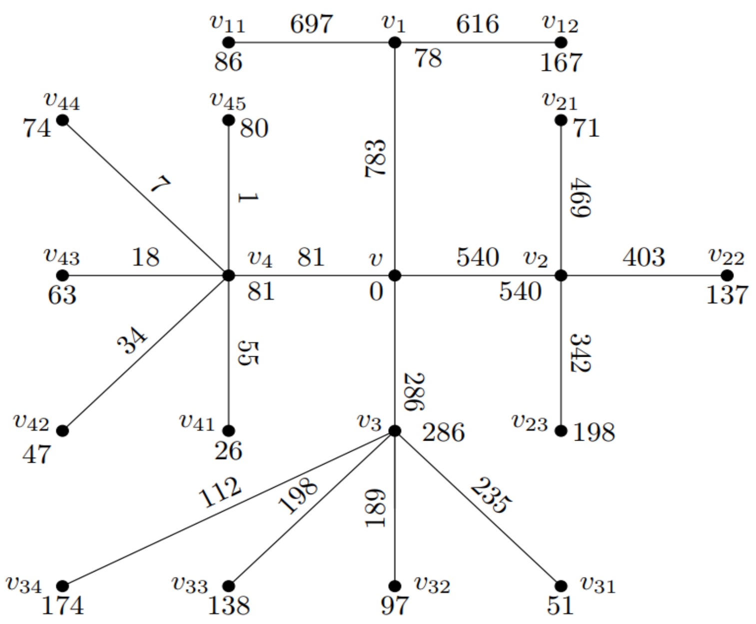


**Fig. 1** St(2,3,4,5)

### *3.2 Theorem*

Coconut tree CT(n, m) Mahendran (2021) is heptagonal graceful for all $n \geq 1$; $m \geq 2$.

**Proof**

Let $G$ be a graph CT(n, m).

Let $V(G) = \{v, v_i, u_j: 1 \leq i \leq n, 1 \leq j \leq m-1\}$ *and*

$E(G) = \{vv_i, vu_j, u_j u_{j+1}; 1 \leq i \leq n, 1 \leq j \leq m-2\}$

$G$ has $n+m$ vertices and $n+m+1$ edges.

Let $q = n+m-1$

Let $f: V(G) \to \{0,1,2, \dots, N_q\}$ be defined as follows.

$f(v) = 0$

$$f(u_j) = \begin{cases} f(u_{j-1}) + N_{q-n-(j-1)} \ if\ j\ is\ odd\ and\ 2 \leq j \leq n-1 \\ f(u_{j-1}) - N_{q-n-(j-1)} \ if\ j\ is\ even\ and\ 2 \leq j \leq m-1 \end{cases}$$

Let $f^*$ be the induced edge labeling of $f$

Then $f^*(vv_i) = N_{q-i+1}, 1 \leq i \leq n$

$f^*(vu_1) = N_{q-n}$

$f^*(u_j u_{j+1}) = N_{q-n-j}, 1 \leq j \leq m-2$

The induced edge labels $N_1, N_2, \dots, N_q$ are distinct and consecutive heptagonal numbers. Hence the coconut tree graph is heptagonal graceful.

#### *3.2.1 Illustration*

Heptagonal graceful labeling of CT (4, 5) is given in Fig. 2;

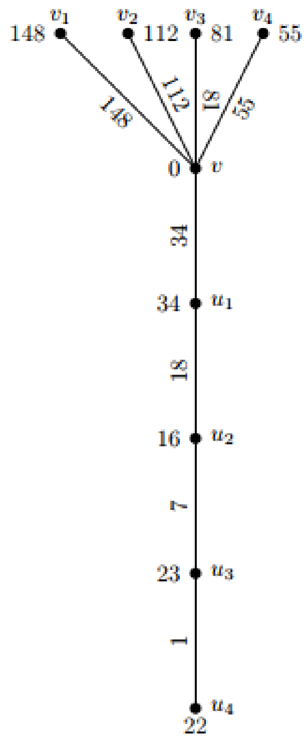


**Fig. 2** CT (4, 5)

***3.3 Theorem***

Banana tree $\mathrm{Bt}(\mathrm{n}_1, \mathrm{n}_2, \ldots, \mathrm{n}_\mathrm{m})$ Mahendran (2021) is heptagonal graceful.

**Proof**

Let $G$ be the graph $Bt(n_1, n_2, \ldots, n_m)$ .

Let $V(G) = \{v, v_i, w_i, w_{ij}: 1 \le i \le m, 1 \le j \le n_i - 1\}$ *and*

$E(G) = \{vv_i, v_iw_i, w_iw_{ij}; 1 \le i \le m, 1 \le j \le n_i - 1\}$

$G$ has $m + n_1 + n_2 + \cdots + n_m + 1$ vertices and $m + n_1 + n_2 + \cdots + n_m$ edges.

Let $q = m + n_1 + n_2 + \cdots + n_m$

Let $f: V(G) \to \{0,1,2, \ldots, N_q\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = N_{q-i+1}, 1 \le i \le m$

$f(w_i) = f(v_i) - N_{q-m-[n_1+n_2+\cdots+n_{i-1}]}, 1 \le i \le m$

$f(w_{ij}) = f(w_i) + N_{q-m-[n_1+n_2+\cdots+n_{i-1}]-j}, 1 \le i \le m, 1 \le j \le n_i - 1$

Let $f^*$ be the induced edge labelings of $f$.

Then $f^*(vv_i) = N_{q-i+1}; 1 \le i \le m$

$f^*(v_iw_i) = N_{q-m-[n_1+n_2+\cdots+n_{i-1}]}, 1 \le i \le m$

$f^*(w_iw_{ij}) = N_{q-m-[n_1+n_2+\cdots+n_{i-1}]-j}, 1 \le j \le n_i - 1, 1 \le i \le m$

The induced edge labels $N_1, N_2, \ldots N_q$ are distinct and consecutive heptagonal numbers.

Hence the banana tree graph is heptagonal graceful.

*3.3.1 Illustration*

Heptagonal graceful labeling of Bt (2,3) is given in Fig. 3;

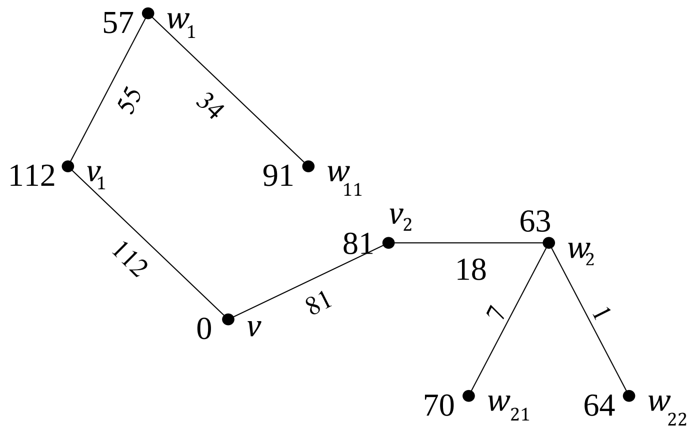


**Fig. 3** Bt (2,3)

***3.4 Theorem***

$K_{1,n} \odot K_1$ Mahendran (2021) is heptagonal graceful.

**Proof**

Let *G* be the graph $K_{1,n} \odot K_1$ .

Let $V(G) = \{v, v_i, u_i, w: 1 \leq i \leq n\}$ *and* $E(G) = \{vv_i, v_iu_i, vw; 1 \leq i \leq n\}$

*G* has $2n + 2$ vertices and $2n + 1$ edges

Let q = 2n + 1

Let $f: V(G) \rightarrow \{0,1,2, \dots, N_q\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = N_{q-(i-1)}$

$f(w) = N_{q-n}$

$f(u_i) = f(v_i) - N_{q-(n+i)}; 1 \leq i \leq n$

Let $f^*$ be the induced edge labeling of $f$.

Then $f^*(v_iu_i) = N_{q-(n+i)}; 1 \leq i \leq n$

$f^*(vw) = N_{q-n}$

$f^*(vv_i) = N_{q-(i-1)}; 1 \leq i \leq n$

The induced edge labels $N_1, N_2, \dots, N_q$ are distinct and consecutive heptagonal numbers.

Hence $K_{1,n} \odot K_1$ is heptagonal graceful.

*3.4.1 Illustration*

Heptagonal graceful labeling of $K_{1,6} \odot K_1$ is given in Fig. 4;

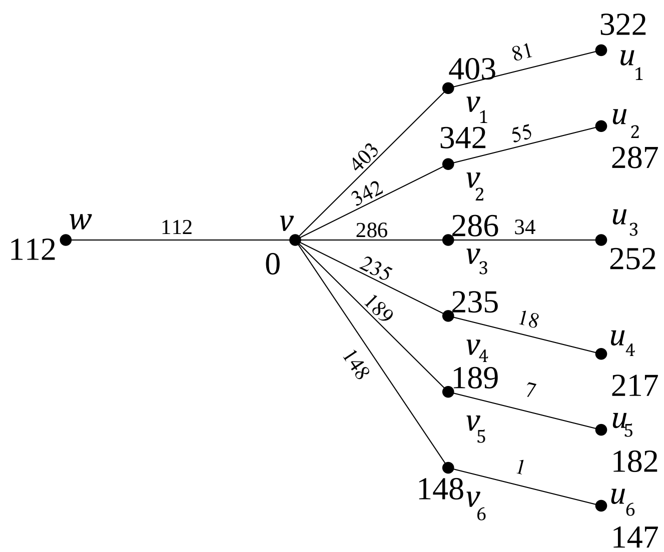


**Fig. 4** $K_{1,6} \odot K_1$

### *3.5 Theorem*

A Y – tree Mahendran (2021) is heptagonal graceful

**Proof**

Let $G$ be the $Y - tree$

Let $V(G) = \{v, v_i: 1 \le i \le n\}$ $and$ $E(G) = \{vv_{i+1}, vv_{n-1}; 1 \le i \le n-1\}$

$G$ has $n + 1$ vertices and $n$ edges.

Let $q = n$

Let $f : V(G) \rightarrow \{0,1,2,\ldots,N_q\}$ be defined as follows.

$f(v_1) = 0$

$$f(v_i) = \begin{cases} f(v_{i-1}) - N_{q-i+2} \; if \; i \; is \; odd \; and \; 2 \le i \le n \\ f(v_{i-1}) + N_{q-i+2} \; if \; i \; is \; even \; and \; 2 \le i \le n \end{cases}$$

$f(v) = f(v_{n-1}) - N_1$

Let $f^*$ be the induced edge labeling of $f$.

Then $f^*(v_i v_{i+1}) = N_{q-i+1}; 1 \le i \le n-1$

$f^*(vv_{n-1}) = N_1$

The induced edge labels $N_1, N_2, \ldots, N_q$ are distinct and consecutive heptagonal numbers.

Hence $Y - tree$ is heptagonal graceful.

#### *3.5.1 Illustration*

Heptagonal graceful labeling of $Y_5$ is given in Fig. 5;

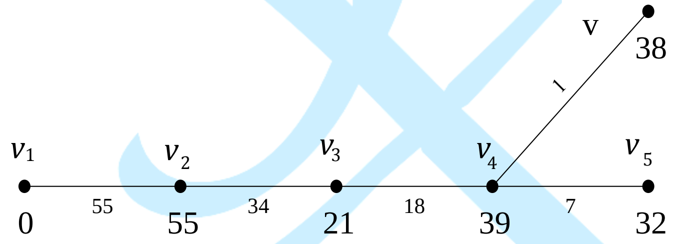


**Fig. 5** *Y – tree* $Y_5$

## 4. Conclusions

We have presented a few new results on Heptagonal graceful labeling on simple graphs. Analogous work can be carried out for other families and in the context of different types of graph labeling technique.

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**Declaration of Conflict**

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