

A Ramp-Type Deterioration EOQ Model with On-Hand Inventory

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Abstract

In this paper, we develop an inventory model for ramp type deteriorations with different demand rate for different parameters. In this proposed model, shortages are not allowed. The model is explained with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to various parameters of the system is given and results are discussed in details.

Keywords: Inventory model, Ramp type deterioration, Demand, Economic order quantity

1. Introduction

Recent years have been a lot of interest in inventory models with degrading items. There has already been enough work completed for inventory control by numerous writers in numerous areas of practical issues. Minimizing the cost of carrying inventory is the primary goal of inventory management. Determining the ideal stock and time for inventory replenishment is crucial for meeting future demand. Choosing when and how much to order in order to keep the cost of the inventory system as low as possible is one of the key concerns of inventory management. Deterioration plays a big part in inventory systems. Over time, the majority of the objects decay. Therefore, it is important to consider the impact of deterioration while deciding on the best inventory management strategy for that category of products.

The demand rate is taken into account in traditional inventory models as a constant. However, the demand for tangible items may be time-, price-, and stock-dependent. An EOQ model with constant demand rate has been established by Goyal (1985) under the circumstances of allowable payment delays. Goyal neglected the discrepancy between the selling price and the purchasing cost in his model. Dave (1985) expanded on Goyal's model by supposing that the selling price must be greater than the cost of acquisition. In 1995, Aggarwal and Jaggi examined the exponentially degrading inventory model in the context of allowable payment delays. In order to account for shortages, Jamal et al.. (2000) further expanded the model. When the supplier allows a payment delay for an order of a good whose demand rate is a function of price elasticity, Hwang and Shinn (1997) established the model for estimating the retailer's optimal pricing and lot size simultaneously.

Teng (2002) adapted Goyal's model by taking into account the fact that the selling price and the purchasing price are not identical (1985). In light of a reasonable payment delay, Huang (2007) looked into the retailer's best restocking strategy. Tripathy and Mishra (2011) created an inventory model on ordering policy

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for linearly degrading items for declining demand with allowable payment delay. If the order quantity is higher than or equal to a certain quantity, the supplier must offer the customer trade credit, according to a model proposed by Chang et al.. (2003). Furthermore, Chang et al.. (2003), Chung and Huang (2009), Teng et al. (2005). It's common knowledge that holding costs in inventory models are constant, although this may not always be the case. Many researchers have created generalised EOQ models to represent diverse holding cost functions, including Goh (1994), Muhlemaan and Valtis Spanopoulos (1980), Weiss (1982), and Naddor (1966). The common element throughout the aforementioned research studies is that there is a complete backlog of unmet demand brought on by shortages. However, in practice, the demand for goods like food, medications, raw materials, etc. is typically lost during the scarcity time. Montgomery et al.. (1973) investigated stochastic and deterministic demand inventory models with a combination of backorder and lot sales. In order to discover the best production inventory control policies, Park (1982) refined and established Mak's (1987) approach by using a uniform replenishment rate. A partially backlogged inventory model was created by Papachristos and Skouri in 2002, and it shows how the backlog rate falls off exponentially as waiting times rise.

The percentage of unmet demand that was back ordered was subsequently expanded by Teng et al.. (2003) to include any decreasing function of the time before the next replenishment. The partial backlogging EOQ model was generalised by Teng and Yang (2004) to incorporate time-varying purchasing cost. On the basis of highest profit, Yang (2005) compared various partial backlogging inventory lot size models for degrading goods. Two price and lot sizing models for degrading goods with shortages were contrasted by Teng et al.. in 2007. An inventory model for degrading goods with a stock-dependent consumption rate and partial backlog shortages was developed by Yang et al.. in 2010. For items that are degrading and have shortages, Dye et al.. (2007) devised inventory and price methods. An inventory model with a general ramp type demand rate, a constant rate of deterioration, a partial backlog of unmet demand, and guidelines for acceptable payment delays was developed by Skouri et al.. in 2011. According to Sana (2010), shortages are acceptable, the deterioration rate is assumed to be time proportionate, and the demand rate is price dependent. Hou (2006), Jagga et al.. (2006), Patra et al.. (2010), Lin (2012), and others have written relevant works on inventory systems with partial backlogs and shortages.

For goods with price-dependent demand, Nita Shah (2012) created a time-proportional deterioration model with replenishment strategy and no shortages. Sarkar (2013) created a degrading model taking into account demand that is reliant on the selling price. Chowdhury and Ghosh (2014) created an inventory model with price- and stock-sensitive demand for decaying goods. Khana et al.. (2017) created a lot size deterioration model for items of subpar quality taking into account price-dependent demand. Zadeh was the first to explain that fuzziness might cause uncertainty in specific circumstances, and he also provided some methods for making decisions in these circumstances (1970). The linearly degrading EOQ model for defective items with price-dependent demand under various fuzzy settings was also developed by Pattnaik et al.. in 2021.

Gupta and Vrat (1986), Mandal and Phaujdar (1989), Baker and Urban (1988), and others are among the significant papers with inventory-level-dependent demand rates that have been published to date. While Mandal and Phaujdar (1989) examined an inventory level, Gupta and Vrat (1986) discussed a scenario in which it was assumed that the demand rate was based on the volume of orders. Baker and Urban (1988) have analyzed a similar situation assuming the demand rate to be dependent on the on-hand inventory i according to the relation $R(i) = \alpha i^\beta$ where $\alpha > 0, 0 < \beta < 1$. Sahu et al.. (2007) have analysed a similar situation, assuming the demand rate to be depend on the on-hand inventory i according to the relation $R(i) = \alpha e^{-\beta i}$ where $\alpha > 0, 0 < \beta < 1$. Yadav et al. (2022) have analyze inventory model for decay items with safe chemical storage and inflation using artificial bee colony algorithm. Again Samal, Mishra and Kalam (2022) have

analysed two situations, assuming the demand rate to be depend on the on-hand inventory i without shortage, according to the relation;

$$R(i) = \alpha i^\beta, \quad i \geq S_0 \text{ and } R(i) = \alpha e^{-\beta i}, \quad 0 \leq i \leq S_0, \quad \alpha > 0, \quad 0 < \beta < 1.$$

The present inventory model makes an attempt to study the situation in which (1) Deterioration rate is constant and ramp type; (2) shortages are not allowed. Several results have been obtained. An optimal solution of the total cost is discussed for various partial backlogging issues. Finally, the sensitivity analysis is given to validate the proposed model.

2. Fundamental Assumptions and Notations of the Model

The basic assumptions made to develop the proposed model are the following.

- (1) Replenishment rate is infinite i.e., replenishment is instantaneous but replenishment size is finite.
- (2) Lead time is zero.
- (3) No shortages are permitted.
- (4) The time horizon is finite.
- (5) The inventory system involves only one item.
- (6) The demand rate is dependent on the on-hand inventory down to a level S_0 , beyond which it is assumed to be constant. The demand rate $R(i)$ of the item when the on hand inventory level is i

$$R(i) = \alpha i^\beta, \quad i \geq S_0$$

$$R(i) + Hi(t) = D, \quad 0 \leq i \leq S_0, \quad \alpha > 0, \quad 0 < \beta < 1, \quad D = \alpha S_0^\beta$$

$$\text{where } H = \begin{cases} 0, & \text{when } 0 < t < t_1 \\ \theta, & \text{when } t > t_1 \end{cases}, \quad 0 < \theta < 1$$

In addition the following notations are used throughout the paper.

s : the selling price

C : unit cost

C_1 : holding cost per unit per unit time

C_3 : replenishment cost per replenishment

S : total inventory

S_0 : inventory level at time t_1

T : total time period

$\pi(S)$: profit per unit time

All the costs, $\alpha, \beta, \theta, S_0$ are known and constant.

3. Basic Equations Governing the Model and Their Solutions

The basic equations governing the present model are the following:

$$\frac{di}{dt} = -\alpha i^\beta, \quad 0 \leq t \leq t_1. \quad (1)$$

$$\frac{di}{dt} + \theta i(t) = -D, \quad t_1 \leq t \leq T. \quad (2)$$

where $t, i, \alpha, \beta, D, t_1, T$ are defined.

The solution of the differential equation (1) using the initial condition $i = S$ at $t = 0$, is

$$i = \left\{ -\alpha(1-\beta)t + S^{1-\beta} \right\}^{\frac{1}{1-\beta}} \quad 0 \leq t \leq t_1 \quad (3)$$

using the condition $i = S_0$ at $t = t_1$,

$$S_0 = \left\{ -\alpha(1-\beta)t_1 + S^{1-\beta} \right\}^{\frac{1}{1-\beta}} \quad (4)$$

$$\text{So } t_1 = \frac{S^{1-\beta} - S_0^{1-\beta}}{\alpha(1-\beta)} \quad (5)$$

The solution of the differential equation (2) using the condition $i = 0$ at $t = 0$, is

$$i = \frac{D}{\theta} \left\{ e^{\theta(T-t)} - 1 \right\}, \quad t_1 \leq t \leq T \quad (6)$$

Using the condition $i = S_0$ at $t = t_1$, we find from (6)

$$S_0 = \frac{D}{\theta} \left\{ e^{\theta(T-t_1)} - 1 \right\}$$

$$t_1 = T - \frac{1}{\theta} \ln \left(\frac{\theta S_0}{D} + 1 \right) \quad (7)$$

From (5) and (7) we get

$$T = \frac{S^{1-\beta} - S_0^{1-\beta}}{\alpha(1-\beta)} + \frac{1}{\theta} \ln \left(\frac{\theta S_0}{D} + 1 \right) \quad (8)$$

$$\text{Now } H = \int_0^T idt = \int_0^{t_1} idt + \int_{t_1}^T idt$$

Substituting the value of i in the above integrals and then integrating, we find;

$$H = \int_0^{t_1} \frac{\ln(-\alpha\beta t + e^{\beta S})}{\beta} dt + \int_{t_1}^T \frac{D}{\theta} \left\{ e^{\theta(T-t)} - 1 \right\} dt$$

Eliminating t_1 and T

$$H = \frac{1-\beta}{2-\beta} \left\{ S_0^{2-\beta} - S^{2-\beta} \right\} + \left\{ -\frac{1}{\theta} \ln \left(\frac{\theta S_0}{D} + 1 \right) - \frac{1}{\theta} + \frac{\theta S_0}{D} + 1 \right\} \quad (9)$$

Now the profit function $\pi(S)$ (profit per unit time) is

$$\pi(S) = \frac{(s-C)S - C_1H - C_3}{T}$$

$$\pi(S) = \frac{(s-C)C_1 \left[\frac{1-\beta}{2-\beta} \left\{ S_0^{2-\beta} - S^{2-\beta} \right\} + \left\{ -\frac{1}{\theta} \ln \left(\frac{\theta S_0}{D} + 1 \right) - \frac{1}{\theta} + \frac{\theta S_0}{D} + 1 \right\} \right] - C_3}{\frac{S^{1-\beta} - S_0^{1-\beta}}{\alpha(1-\beta)} + \frac{1}{\theta} \ln \left(\frac{\theta S_0}{D} + 1 \right)} \quad (10)$$

The necessary condition for $\pi(S)$ to be a maximum is

$$\frac{d\pi(S)}{dS} = 0$$

$$\Rightarrow S^{2-2\beta} \left\{ C_1\theta(1-\beta)(2-\beta) \right\}$$

$$+ S^{1-\beta} \left\{ (s-C)(2-\beta)\theta + C_1\alpha(1-\beta)^2(2-\beta) \ln \left(\frac{\theta S_0}{D} + 1 \right) - \theta(1-\beta)(2-\beta)C_1S_0^{1-\beta} \right\}$$

$$- S^{1+\beta} \left\{ (s-C)(1-\beta)(2-\beta)\theta \right\}$$

$$+ S^\beta \left\{ \begin{aligned} &C_1\alpha(1-\beta)^2\theta S_0^{2-\beta} + C_1(1-\beta)(2-\beta) - \theta(1-\beta)(2-\beta)C_1 \left(\frac{\theta S_0}{D} + 1 \right) \\ &- C_3\theta(1-\beta)(2-\beta) + C_1(1-\beta)(2-\beta) \ln \left(\frac{\theta S_0}{D} + 1 \right) \end{aligned} \right\}$$

$$-S^2\{C_1\theta(1-\beta)^2\} + (s-C)(2-\beta)\left\{\alpha(1-\beta)\ln\left(\frac{\theta S_0}{D} + 1\right) - \theta S_0^{1-\beta}\right\} = 0 \quad (11)$$

$$T = \frac{S^{1-\beta} - S_0^{1-\beta}}{\alpha(1-\beta)} + \frac{1}{\theta} \ln\left(\frac{\theta S_0}{D} + 1\right) \quad (12)$$

The roots of the equation (11) give the global maximum for the profit function $\pi(S) = \pi(S^*)$ occurs at $S = S^*$ and $T = T^*$. Such a positive root S of equation (11) for which $\frac{d^2\pi(S)}{dS^2} < 0$ gives a local maximum for the profit function $\pi(S)$.

4. Numerical Examples

To illustrate the model developed, the following one numerical example have been considered. The numerical solution of these equations is obtained by using the software MATLAB

We choose $\alpha = 0.6$, $\beta = 0.2$, $\theta = 0.01$, $s = 300$, $C = 14$, $S_0 = 8$, $C_3 = 12$, $C_1 = 0.5$, Using the decision rule, we get $S = 10.058$, $T = 10.641$ and $\pi(S) = 274.69$.

5. Sensitivity analysis and observation

For various associated parameters, a sensitivity analysis is presented below in tabular and graphical form.

1 Table 1 shows the outcome of

Table 1 Optimal solution for distinct values of α and ($\beta = 0.2, \theta = -0.01, s = 300, C = 14, S_0 = 8, C_3 = 12, C_1 = 0.5$)

α	S	$\pi(S)$	$T(S)$
0.4	10.864	186.08	16.969
0.5	10.384	230.43	13.097
0.6	10.058	274.69	10.641
0.7	9.8213	318.9	8.95
0.8	9.6423	363.08	7.7177

Table 2 Optimal solution for distinct values of β and ($\alpha = 0.6, \theta = -0.01, s = 300, C = 14, S_0 = 8, C_3 = 12, C_1 = 0.5$)

β	S	$\pi(S)$	$T(S)$
0.1	12.31	223.89	15.985
0.15	10.849	248.14	12.706
0.2	10.058	274.69	10.641
0.25	9.5695	303.97	9.1495
0.3	9.2423	336.33	7.9868

Table 3 Optimal solution for distinct values of θ and ($\alpha = 0.6, \beta = 0.2, s = 300, C = 14, S_0 = 8, C_3 = 12, C_1 = 0.5$)

θ	S	$\pi(S)$	$T(S)$
0.005	10.079	274.81	10.841
0.01	10.058	274.69	10.641
0.015	10.552	277.44	10.989
0.02	11.155	280.66	11.454
0.025	11.786	283.89	11.948

Table 4 Optimal solution for distinct values of s and ($\alpha = 0.6, \beta = 0.2, \theta = -0.01, C = 14, S_0 = 8, C_3 = 12, C_1 = 0.5$)

s	S	$\pi(S)$	$T(S)$
100	10.781	85.607	11.395
200	10.239	180.17	10.83
300	10.058	274.69	10.641
400	9.967	369.21	10.545
500	9.9127	463.72	10.488

Table 5 Optimal solution for distinct values of C and ($\alpha = 0.6, \beta = 0.2, \theta = -0.01, s = 300, S_0 = 8, C_3 = 12, C_1 = 0.5$)

C	s	$\pi(S)$	$T(S)$
10	10.053	278.47	10.636
12	10.055	276.58	10.638
14	10.058	274.69	10.641
16	10.06	272.8	10.643
18	10.062	270.91	10.646

Table 6 Optimal solution for distinct values of S_0 and ($\alpha = 0.6, \beta = 0.2, \theta = -0.01, s = 300, C = 14, C_3 = 12, C_1 = 0.5$)

S_0	s	$\pi(S)$	$T(S)$
6	7.5104	258.63	8.4737
7	8.7704	267.03	9.5618
8	10.058	274.69	10.641
9	11.371	281.77	11.712
10	12.708	288.37	12.777

Table 7 Optimal solution for distinct values of C_3 and ($\alpha = 0.6, \beta = 0.2, \theta = -0.01, s = 300, C = 14, S_0 = 8, C_1 = 0.5$)

C_3	s	$\pi(S)$	$T(S)$
10	10.091	274.88	10.676
11	10.074	274.79	10.658
12	10.058	274.69	10.641
13	10.041	274.6	10.623
14	10.024	274.51	10.606

Table 8 Optimal solution for distinct values of C_1 and ($\alpha = 0.6, \beta = 0.2, \theta = -0.01, s = 300, C = 14, S_0 = 8, C_3 = 12$)

C_1	s	$\pi(S)$	$T(S)$
0.1	9.8354	272.48	10.407
0.3	9.9475	273.59	10.525
0.5	10.058	274.69	10.641
0.7	10.165	275.79	10.754
0.9	10.271	276.88	10.865

- (i) Observation from Table-1: The inventory S and the time $T(S)$ decreases with increase in the value of α where as the profit $\pi(S)$ increases.
- (ii) Observation from Table-2: The inventory S and the time $T(S)$ decreases with increase in the value of β where as the profit $\pi(S)$ increases.

- (iii) Observation from Table-3: The inventory S , the time $T(S)$ and the profit $\pi(S)$ increases with increase in the value of θ .
- (iv) Observation from Table-4: The inventory S and the time $T(S)$ decreases with increase in the value of s where as the profit $\pi(S)$ increases.
- (v) Observation from Table-5: The inventory S and the time $T(S)$ increases with increase in the value of C where as the profit $\pi(S)$ decreases.
- (vi) Observation from Table-6: The inventory S , the time $T(S)$ and the profit $\pi(S)$ increases with increase in the value of S_0 .
- (vii) Observation from Table-7: The inventory S , the time $T(S)$ and the profit $\pi(S)$ decreases with increase in the value of C_3 .
- (viii) Observation from Table-8: The inventory S , the time $T(S)$ and the profit $\pi(S)$ increases with increase in the value of C_1 .

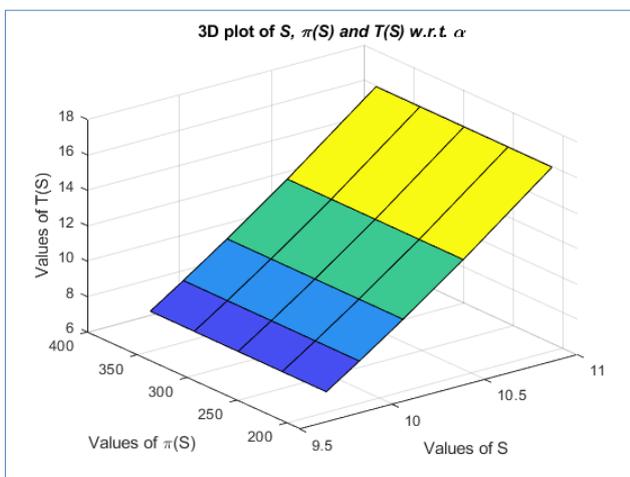


Fig. 1 Graph of $S, \pi(S), T(S)$ with respect to α

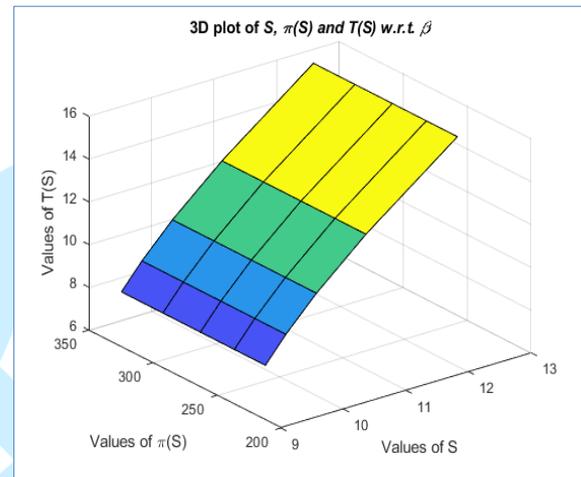


Fig. 2 Graph of $S, \pi(S), T(S)$ with respect to β

6. Conclusion

Among the significant papers with inventory-level-dependent demand rates that have been published thus far. The present inventory model has two sorts of demand, the first of which is dependent on stock level and the second of which has a continuous demand. Analyzed are models where the demand rate depends on the starting stock level as well as demand rate dependent models. The stock level's motivational impact on customers is based on the current stock level. The model can be applied to the inventory control of specific items that degrade with time, such as food items, electronic components, trendy goods, and others. Due to such criteria as goodwill, good quantity, authentic pricing level, location of the shop, good quality of the things, etc., only a small number of customers attend to purchase goods. Based on the ideal solution, we also have some significant outcomes. From managerial point of view, sensitivity analysis is quite sensitive with respect to variation of parameters.

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